Anoka-Hennepin Secondary Curriculum Unit Plan

Department:	Mathematics	Course:	Intermediate Algebra	Unit 5 Title:	Solving Quadratic Equations	Grade Level(s):	9
Assessed Trimester:	Trimester B	Pacing:	15-16 Days	Date Created:	6/13/2013	Last Revision Date:	8/27/2014

Course Understandings: *Students will understand that:*

A. Relationships exist between real-world situations, mathematical equations, inequalities and graphs for linear, exponential, absolute value, radical, and polynomial functions.
B. Equations and inequalities can be categorized by form and that each form has specific processes to consider when solving and graphing.
C. There are a variety of strategies of varying efficiency for simplifying linear, absolute value, exponential, radical, complex and polynomial expressions.
D. The complex number system is an essential extension of the real number system for the manipulation of all quadratic functions.
E. The context of a problem is important in recognizing the reasonableness of a solution.
F. There are benefits and limitations in the use of calculators and other technology to solve mathematical situations.

DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

Established Goals

Minnesota State/Local/Technology Standard(s) addressed (2007):

- **Standard (9.2.3.# - Modified):** Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.
  Benchmark:
  9.2.3.3 Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares.
  9.2.3.5 Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers.

- **Standard (9.2.4.# - Modified):** Represent real-world and mathematical situations using equations and inequalities involving quadratic functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.
  Benchmark:
  9.2.4.1 Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.
  9.2.4.3 Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.

Transfer

Students will be able to independently use their learning to: (product, high order reasoning)

- Model, analyze and solve real world situations using quadratic functions.

Meaning

Unit Understanding(s):	Essential Question(s):
Students will understand that: • A real-world situation can be represented as a quadratic function and will demonstrate an understanding of how to find reasonable solutions.	**Students will keep considering:** • How do I decide the best method to solve a quadratic equation? • What real world questions can I answer using the solutions of a quadratic equation?

Acquisition

Knowledge - Students will:
- Determine how many solutions can a quadratic equation have.
- Demonstrate understanding of the relationship between solutions of quadratic equations and their graphs.

Reasoning - Students will:
- Interpret a solution in the original context.
- Draw qualitative conclusions based on the graphs and equations.
- Evaluate the reasonableness of the solution based on the context of the problem.
- Relate the solution(s) of a quadratic equations to its real world situation.

Skills - Students will:
- I can graph quadratic inequalities
- Demonstrate understanding of how to find real and non-real solutions of quadratic equations for real-world situations.
- Solve by factoring, finding square roots, completing the square and the quadratic formula.
- Demonstrate understanding of simplifying the solution of a quadratic equation.
  - Simplify a radical expression (including those that create imaginary numbers)
- Verify that an answer is a solution.
- Add and subtract radical expressions (including those with imaginary numbers).
- Multiply radical binomials (including those that contain complex numbers).
- Determine the number of real and nonreal solutions for a quadratic equation.
  - Find by factoring, using the discriminant, solving the equation and using the graph.
- Represent relationships using quadratic inequalities and find solutions.
- Write a quadratic model for a real world situation.

Common Misunderstandings
- Students mistake i for a variable.
- Students will compute the product of two complex numbers and not write the result as a complex number (i.e. $(2 + i)(3 - 2i) = 6 - i - 2i^2$)
- Although this answer is not wrong, the answer does not emphasize that the set of complex numbers is closed under the operation of multiplication.
- Students will simplify square roots of negative numbers as $\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5} = i\sqrt{5}$. Although this answer is correct the use of the radical notation is not correct. The rule $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ only works when a ≥ 0 and b ≥ 0. This misuse of the property leads to wrong answers when students evaluate expressions like $\sqrt{-3} \cdot \sqrt{-12} = \sqrt{36} = 6$. The expression

$$\begin{aligned} &\sqrt{-3} \cdot \sqrt{-12} \\ &= i\sqrt{3} \cdot i\sqrt{12} \\ &= i^2\sqrt{36} \\ &= -6 \end{aligned}$$

- Students will incorrectly add radical expressions with unlike radicands (i.e. $\sqrt{3} + \sqrt{2} = \sqrt{5}$).
- Students often struggle to create a mathematical model for a real-world situation.
- Students incorrectly apply the distributive property to multiply polynomials. (e.g., $(3a + b)^2 = (3a)^2 + (b)^2$).
- Students will omit 0 from the solution set. *For example,* when students are asked to solve the equation $2x^2 - 10x = 0$, they may factor to get $2x(x - 10) = 0$, then either divide both sides of the equation by 2x or just focus on the factor (x - 10) to state that the solution is only $x = 10$.
- Many students who solve quadratic equations by taking the square root of both sides of the equation will lose one of the solutions. *For example,*

$$(x+2)^2 = 9 \quad \rightarrow \quad \sqrt{(x+2)^2} = \sqrt{9} \quad \rightarrow \quad x + 2 = 3 \quad \rightarrow \quad x = 1$$

- Students will turn an expression into an equation by setting it equal to zero and solving.
  For example, factor the trinomial $m^2 + m - 20$.
  Student writes:

$$\begin{aligned} &m^2 + m - 20 \\ &(m + 5)(m - 4) \\ &(m + 5)(m - 4) = 0 \\ &m = -5 \text{ or } m = 4 \end{aligned}$$

- Students may find non-real complex numbers to be difficult to simplify, or may not understand what a non-real solution represents in a given quadratic context.
- Students may not take the real-world context for a quadratic relationship into account when giving solutions.

Essential new vocabulary
- Completing the square
- Complex numbers
- Discriminant
- Exponential functions
- Factoring
- Imaginary number
- Intercepts
- Irrational numbers
- Quadratic equation
- Quadratic formula
- Rational numbers
- Real numbers
- Roots of a function
- Solutions of a functions
- Zeroes of a function
- Zero product property